

HW 3

12

Box with 7 red and 13 blue balls.



Take 2 balls out of the box and discard them without looking.

Then draw a 3rd ball and you notice its red. What's the prob. the two discarded balls are blue?

Let BB, BR, RR be the events that the first two balls were blue/blue, blue/red, or red/red.

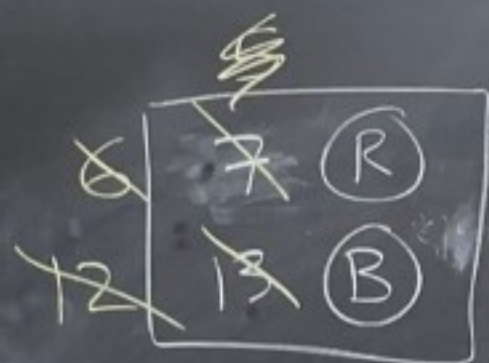
Let R be the event the 3rd ball is red.

We want $P(BB|R)$

$$P(BB|R) = \frac{P(BB\bar{R})}{P(R)} = \frac{P(R|BB) \cdot P(BB)}{P(R)}$$

$$P(BB\bar{R}) = P(R|BB) \cdot P(BB)$$

$$P(R|BB) = \frac{P(R \cap BB)}{P(BB)} = \frac{P(BB\bar{R})}{P(BB)}$$



$$P(BB) = \frac{\binom{13}{2}}{\binom{20}{2}} = \frac{13 \cdot 12}{2} = \boxed{\frac{78}{190}}$$

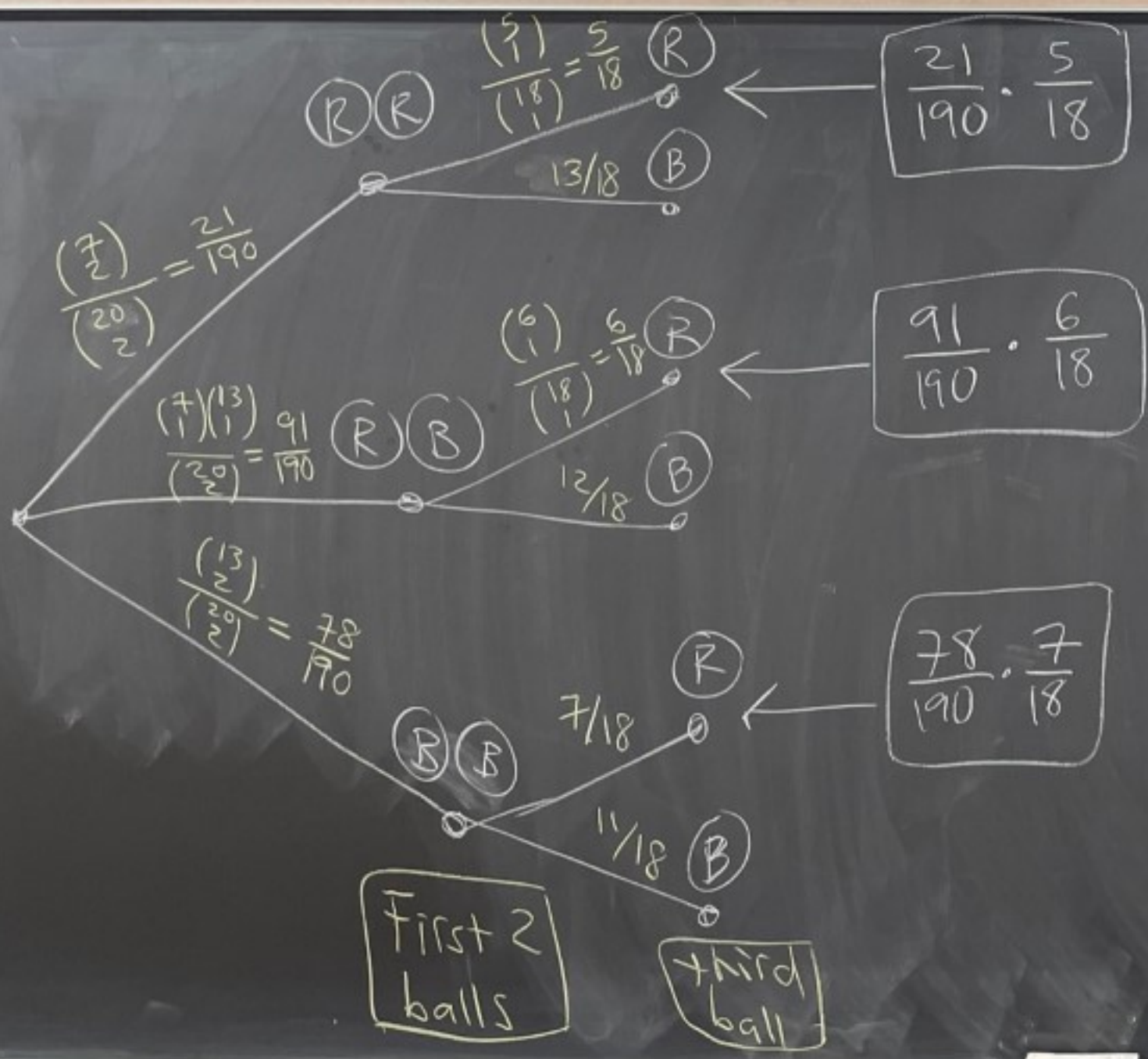
$$\binom{13}{2} = \frac{13!}{2! \cdot 11!} = \frac{13 \cdot 12 \cdot \cancel{11!}}{2! \cdot \cancel{11!}} = \frac{13 \cdot 12}{2}$$

$$\binom{n}{2} = \frac{n(n-1)}{2} \quad \star$$

$$P(R|BB) = \frac{\binom{7}{1}}{\binom{18}{1}} = \boxed{\frac{7}{18}}$$

after 2 blues
taken out have





$$\frac{P(R|BB) \cdot P(BB)}{P(R)} = P(BB|R)$$

$$P(R) = \frac{21}{190} \cdot \frac{5}{18} + \frac{91}{190} \cdot \frac{6}{18} + \frac{78}{190} \cdot \frac{7}{18}$$

$$= \frac{7}{20}$$

Answer

$$P(BB|R) = \frac{P(R|BB) \cdot P(BB)}{P(R)} = \frac{(7/18)(78/190)}{7/20}$$

$$= \frac{26}{57} \approx 0.456$$